STOCHASTIC SIMULATION APPROACH TO DEBT SUSTAINABILITY ANALYSIS: APPLICATION TO ZIMBABWE

BY

NEBSON MUPUNGA
PRINCIPAL ECONOMIST
RESERVE BANK OF ZIMBABWE
ABSTRACT

This paper applies dynamic stochastic simulation methods to assess medium to long-term public debt sustainability in Zimbabwe and provides probability measures for projections of public debt burden. The methodology applied involves estimating a fiscal reaction function and using it to simulate public debt path using a stochastic approach and historical information on drivers of public debt accumulation and their volatility. The results from the baseline scenario show that Zimbabwe’s public debt would not deviate much from the desired indicative public debt benchmarks in the medium to long-term. The risk to the public debt projection is however high with a 47 per cent probability of public debt exceeding the desired threshold in the medium to long term. The policy implication is the need for policy makers to proactively respond to the changing macroeconomic environment and to implement countercyclical fiscal policies to limit the probability of debt from exploding.

Key words: Public Debt Dynamics, Debt sustainability, Stochastic Simulation, fiscal reaction function
SECTION ONE: INTRODUCTION

The sustainability of public debt cannot be determined with certainty. What matters is the risk caused by significant public debt accumulation, than the expected evolution of the public debt. A key determinant of the risk indicator is the quality of government policies in controlling public debt in the event of adverse shocks (Casper van Ewijk, Jasper Lukkezen, Hugo Rojas-Romagosa, 2013). Insight into the risk caused by significant rise in public debt and policy responses is critical as it informs decisions concerning the need for reform. As such the need for a dynamic, forward-looking framework to assess the stability of public debt under uncertainty is of critical importance.

Analysis of public debt under uncertainty is particularly crucial to Zimbabwe given the existence of a public debt overhang in an uncertain macroeconomic environment. The country’s susceptibility to exogenous shocks increased following the adoption of the multi-currency regime in 2009. The adoption of the multiple-currency regime implies that Zimbabwe ceased to have a currency of its own. As a result, the dollarisation phenomenon curtailed the country’s ability to influence policy through traditional demand management tools, such as the discount window and open market operations. In the absence of seigniorage revenue, the only policy instrument available to government to stimulate economic activity and to respond to macroeconomic shocks is external borrowing. Excessive reliance, exacerbates the economy’s vulnerability to external shocks. The current account deficit, which is a major driver of external debt dynamics has also been widening over the period 2000 to 2013, averaging more than 20 percent of GDP. The current account deficit has key been financed by debt creating flows, thus, worsening the already precarious debt overhang with adverse effects on debt sustainability. Unlike, in the past were the country could retire domestic debt through seigniorage revenue, under the multicurrency regime, Zimbabwe has to boost its revenue collections to be able to service both domestic and external debt. This paper is, therefore, motivated by the need to identify how the government has been responding to public debt developments with a view to determine sustainable public debt management policies in the medium to long-term. In doing so, the paper seeks to provide insights into the following key questions:

1. What should be the medium to long-term public debt path given the nature of shocks facing the country and its fiscal policy stance?
2 What should be the government’s fiscal response to ensure medium to long term sustainability of public debt levels?

Answers to these questions will enable government to come up with public debt management policies that are consistent with maintaining public debt at sustainable levels. However, since future borrowing involves uncertainty, this requires stochastic simulation of public debt to obtain the distributions of public debt as well as the median public debt path and the probability of debt deviating from the optimal steady state.

The broad objective of this paper is to simulate the medium term public debt path for Zimbabwe, with a view to craft sustainable public debt policies. Specifically, the paper is aimed at:

i. Analysing the public debt dynamics in Zimbabwe and their implications on sustainable public debt policy;

ii. Identifying the macroeconomic and financial vulnerabilities that might derail public debt from a sustainable path; and

iii. Determine the stochastic public debt path consistent with fiscal policy stance and macroeconomic shocks affecting the country.

Although the International Monetary Fund (IMF) regularly conducts debt sustainability analysis to assess the future path of public debt in Zimbabwe, the analysis has mainly been deterministic and does not take into consideration uncertainty in the public debt portfolio. The debt sustainability analysis, as typically carried out by the IMF has focused instead on debt dynamics. Under this approach, the government can be considered to be operating within its budget constraint as long as the expected fiscal policy stance keeps the debt-to-GDP ratio on a stable (or declining) path.

As pointed out by Celasun, Debrun, and Ostry (2006), there are serious shortcomings to this approach. First, the conditions under which the debt-to-GDP ratio behaves over time are not deterministic but stochastic. The government may have control over its policy setting, but the debt path also depends on macroeconomic conditions that are outside of its control, i.e., GDP growth, interest rates and the exchange rate. Second, even if debt is declining, a high level of debt and its rollover create a risk that liquidity (or other) shocks will unravel into a debt
crisis. Without a handle on the probability distribution of shocks, it is impossible to assign a probability to this risk. This paper derives "fan charts" to depict the probability distribution of the public debt to GDP ratio under a medium-term adjustment scenario, as a result of shocks to GDP growth and interest rates. The distribution of shocks is derived from the past shocks to these variables and the related variance covariance.

The high dependence of Zimbabwe on primary commodities and exposure to terms of trade shocks underscore the need for a stochastic analysis of the dynamics of government primary balance and its response to public debt. Since borrowing decisions are mainly determined by fiscal policy, an understanding of the determinants of fiscal performance is also critical to the formulation of sustainable public debt management policies. Insight into the sustainability of public debt is also essential to policymakers as it creates the need for speedy fiscal consolidation, the need for reform and the determination of the appropriate risk premium on public debt.

The analysis generates a distribution of simulated public debt paths, which shows the effect of fiscal responses and interest and growth rate volatility on public debt to GDP ratios. The results from the stochastic simulations would indicate the ability of government to control its finances in the medium to long-term, thereby allowing government to craft sustainable public debt management policies. If the simulated public debt is large, the government would be considered incapable of controlling its finances and the risk of medium to long-term default would be high. The projected median public debt levels from the simulation analysis, thus, offer a dynamic, forward-looking assessment of the sustainability of public debt.

The rest of the paper is structured as follows. Section two provides a review of Zimbabwe’s public debt dynamics. Section three reviews the theoretical literature on stochastic public debt, fiscal response function and public debt dynamics. Section four describes in detail the methodology for stochastic public debt projections based on the estimated fiscal reaction function and debt dynamics. Section five provides analysis of results while section six concludes and provides policy implications and recommendations.
SECTION TWO: REVIEW OF ZIMBABWE'S PUBLIC DEBT DYNAMICS

This section provides an analysis of Zimbabwe’s public debt from 1980 up to 2012. Zimbabwe presents a very relevant case study among the low income countries (LICs) to analyse debt management policies given the existence of a debt overhang that has been cited as a major hindrance to the country’s economic growth (IMF, 2012). Zimbabwe achieved both periods of debt stability and instability during the three decades under review. The analysis involves decomposing the public debt by creditor type, currency and instrument-type and their implications on maintenance of public debt sustainability.

Assessment of a country’s historical fiscal performance as a proxy for future fiscal performance assists in the formulation of public debt management policy. As highlighted by Reinhart et al. (2003), the track record of a country’s fiscal performance facilitates the assessment of what constitutes an optimal public debt policy. The Reinhart view was based on the fact that a country’s record at meeting its public debt obligations and managing its macro economy in the past is relevant to understanding its ability to sustain moderate to high levels of indebtedness in the medium to long-term. An analysis of historical public debt trends assist in identifying whether fiscal authorities should change the structure of the existing public debt dynamics in a more efficient manner. The analysis of historical public debt dynamics also provides a benchmark against which alternative public debt strategies can be tested and evaluated.

2.1 Evolution of debt sustainability concerns

The debt sustainability concerns of Zimbabwe’s public debt can be traced back to the country’s independence in 1980 when the government embarked on a borrowing spree both domestic and external to address colonial vestiges. The new government adopted a socialist ideology in which expenditure on human and social needs were given prominence. The government was under immense pressure to finance post war reconstruction projects as well as fulfilling the post-independence development agenda that included free education, health and higher wages. This policy stance resulted in increased government expenditure which was not commensurate with the country’s revenue base. As a result, the country began to record high fiscal deficits.
The country’s fiscal position was further exacerbated by recurring droughts which hit the country between 1983 and 1985. Under these circumstances, the government had to commit its limited resources towards drought mitigation programmes. This development added more pressure to the fiscus and increased the government’s borrowing need. The high borrowing requirement resulted in the rapid growth in the Zimbabwe’s public debt. Consequently, the proportion of interest payments on public debt also increased, thereby creating a vicious cycle in debt dynamics. On average, the government was spending about 8.2 per cent on interest expenditures in the 1980s (GoZ, 1990). By 1990, interest expenditure accounted for about sixteen per cent of the total government revenue, or an equivalence of six per cent of the country’s GDP (GoZ, 1990).

The government expenditure continued to rise, particularly in 1998 and 1999 due to government’s need to fulfill regional peace and security commitments in Democratic Republic of Congo (DRC) (Jones, 2011). Against this background, the economy began to experience negative economic growth rates of -0.8 per cent in 1998 and -2.1 per cent in 1999. (GoZ, 2000). Consequently, the government’s fiscal deficit increased from 5.4 per cent in 1997 to 24.6 per cent in 1999 (GoZ, 2000). Furthermore, the shrinking tax base triggered by reduced economic activities, expenditure overruns and huge domestic interest expenditure all contributed to the weakening of the country’s fiscal position. Figure 2.1 below shows the trend in the government primary balance from 1980 to 2012.

**Figure 2.1: Trend in government primary balance**
As shown in Figure 2.1, Zimbabwe recorded extensive primary deficits between 1997 and 1999, which reflects unbudgeted compensation of war veterans and involvement of the government in the DRC. The financing of these expenditures was instrumental in undermining the country’s economic growth prospects and public debt dynamics (Jones, 2011). This era marked the beginning of Zimbabwe’s economic problems with adverse consequences on its debt sustainability.

2.2 Public debt trend in Zimbabwe

Zimbabwe’s public debt-to-GDP ratio was 73 per cent at the end of 2012. This ratio exceeds the internationally recognized benchmark for sustainable public debt levels in developing economies of at most 40 per cent of GDP (IMF 2012). The public debt ratio averaged 80.1 per cent between 1980 and 2012. The public debt ratio reached an all-time high of 105.9 per cent in December 2008. The public debt-to-GDP ratio is generally used by investors to measure a country’s ability to make future payments on its debt. Figure 2.2 below shows the trend in Zimbabwe’s public debt-to-GDP ratio from 1980 to 2012.

Figure 2.2: Public Debt-to-GDP ratio in Zimbabwe (1980-2012)
From Figure 2.2, it can be seen that, Zimbabwe’s public debt was reasonably stable between 1995 and 2000. This was at a time when the economy was still recording fair growth rates. The trend, however, reversed from the year 2000 onwards. The debt level spiraled due to more domestic borrowing in the absence of external support and capitalization of interest payment arrears on external debt. The growth in public debt from 2000 onwards also reflects the impact of penalty charges on external payment arrears (Jones, 2011). The adoption of the multi-currency resulted in high growth rates, which improved the country’s capacity to repay debt. This resulted in a gradual decline in the debt-to-GDP ratio even if the country was not fully servicing its outstanding debt. The decline in the public debt over the period 2009 to 2012 may be attributed to the increases in real GDP, instead of repayment of outstanding debt.

The retrospective view of Zimbabwe’s public debt suggests that it increased during times of economic recession and slowed down during the post crisis period. It is apparent that public debt increased sharply between 2000 and 2008 when the economy experienced an economic crisis characterized by spiraling hyperinflation. Public debt however, maintained a downward trend following the adoption of the multi-currency system in 2009, which was instrumental in fostering economic growth (IMF, 2012).
SECTION THREE: REVIEW OF LITERATURE

An extensive literature that analyses and proposes different measures of public debt sustainability exists. Most of this literature, however, relies on unit root and co integration tests often in combination with the inter-temporal budget constraint to analyze the sustainability of public debt. Bohn (2007), however, shows that the consistency with the inter-temporal budget constraint is not a sufficient condition for public debt sustainability. According to Bohn, it is possible to satisfy the inter-temporal budget constraint, while simultaneously having a mildly explosive path of public debt to GDP ratios. The theoretical framework, motivated by Bohn (1998, 2008), also advocates for the existence of a fiscal reaction function, which implies that the primary balance is positively correlated with lagged public debt levels.

Ghosh et al (2011), however, suggest that the size of primary balance response may vary with the level of the public debt ratio, reacting more strongly when the debt ratio exceeds a given threshold, but then the responsiveness eventually begins to weaken, and then actually decreases at very high public debt levels. Burger (2012) extended the model and calculated stable public debt positions based on the premise that a sustainable public debt policy is the one which stabilizes public debt at whatever level. Recent studies have also illustrated the existence of fiscal fatigue, whereby the government’s ability to increase primary balances cannot keep pace with rising public debt. As a result, the government faces an endogenous public debt limit beyond which public debt cannot be rolled over (Ghosh et al., 2013).

Bohn (2008), approach, applied the inter-temporal budget constraint for government debt and a behavioural equation for the government’s primary balance to analyze the behaviour of public debt. Bohn’s approach equates fiscal sustainability with the stationarity of the public debt-to-GDP ratio and suggests that when the public debt-to-GDP ratio is stationary over time without a trend, one can consider public debt to be sustainable. The starting point of Bohn’s analysis is, therefore, the inter-temporal budget constraint, which states that debt is a function of past debt and interest payments on previous debt outstanding. This is algebraically illustrated as follows:

\[ D_t = (1 + i_t)D_{t-1} - PB_t + SF_t \]  

(1)
Where \( D_t \) is the outstanding debt at time \( t \), \( PB_t \) is the primary balance at time \( t \), \( i_t \) is the nominal interest rate at time \( t \), and \( SF_t \) is the stock-flow adjustment that ensures consistency between net indebtedness and variation in the observed public debt stock. The stock-flow adjustment includes a number of variables, such as the recognition of contingent liabilities, extra budgetary expenditures and other statistical discrepancies. Dividing equation 1 by nominal GDP gives the following:

\[
\frac{D_t}{P_tY_t} = \frac{(1+i_t)}{(1+\pi_t)(1+r_t)} \times \frac{D_{t-1}}{P_{t-1}Y_{t-1}} - \frac{PB_t}{P_tY_t} + \frac{SF}{P_tY_t}
\]  

(2)

Where the nominal GDP is algebraically defined as: \( R_Y_t = (1 + \pi_t)(1 + r_t)P_{t-1}Y_{t-1} \), where \( Y_t \) is the nominal GDP at time \( t \), \( P_t \) is the GDP deflator at time \( t \), \( \pi_t \) is the inflation rate at time \( t \), \( r_t \) is the real interest rate at time \( t \) and \( g_t \) is the real growth rate of the economy at time \( t \). Assuming that \( SF_t = 0 \) the equation can translate to:

\[
d_t = \frac{(1+i_t)}{(1+\pi_t)(1+g_t)}d_{t-1} - pb_t = d_t = \frac{(1+r_t)}{(1+g_t)}d_{t-1} - pb_t
\]  

(3)

Where, the nominal interest rate is given by \( i_t = (1 + r_t)(1 + \pi_t) - 1 \), Defining equation 3 in lower cases result in the following equation:

\[
d_t = \emptyset_t d_{t-1} - pb_t
\]  

(4)

Where \( \emptyset_t = \frac{(1+i_t)}{(1+\pi_t)(1+g_t)} = \frac{(1+r_t)}{(1+g_t)} \) \( d_t = \frac{D_t}{P_tY_t} \) and \( pb_t = \frac{PB_t}{P_tY_t} \)

The parameter \( \emptyset_t \) in equation 4 is known as the automatic debt dynamics, and it can result in the accumulation of public debt without the government contracting any new debt. As shown in the equation, changes to automatic public debt dynamics are explained by the real interest rate and growth rate of the economy. The other determinants of the change in the public debt ratio as shown in equation 2 are the primary balance and the stock flow adjustment, which is a residual. The primary balance is controlled by fiscal policy makers, while interest rates largely depend on actions of monetary authorities. The growth rate \( g_t \) enters into this equation because a higher growth rate tends to reduce the public debt ratio, by raising the denominator of the public debt to GDP ratio.

The key reason for analysing the public debt dynamics is to determine whether public debt is stable or explosive. Accordingly, from equation 4 it can be deduced that for the ratio of the debt to GDP (\( d_t \)) to converge to a predetermined optimal level, \( \emptyset_t < 1 \) or \( r_t < g_t \). However,
if $\emptyset_t > 1$ or $r_t > g_t$, the public debt portfolio would explode from the predetermined optimal path. This condition has increasingly become to be known as the Aaron condition (Aaron 1996). This implies that if the interest rate being paid on debt is greater than the growth rate of the economy, the interest burden on existing debt increases, while the debt to GDP ratio also increases. Subtracting $d_{t-1}$ from both sides of equation 3, results in the following equation:

$$\Delta d_t = (r_t - g_t)d_{t-1} - pb_t$$

Equation 5 shows that changes in the public debt ratio can be decomposed into three factors and their underlying processes that determine the evolution of public debt-to-GDP ratio. Equation 5 shows that for public debt to remain stable ($\Delta d_t = 0$), the primary balance needs to at least cover the interest payments due. However, if past debts are very large or if interest rates are very high, the government would either be required to raise the primary balance or the public debt will increase every year in a snowballing effect because the portion of the payments that cannot be covered by the primary balance will be covered by issuing new loans, thereby increasing the debt stock.

The second part of Bohn’s methodology, involves estimating the fiscal reaction function, which indicates whether the government increases its primary balance in response to changes in the public debt-to-GDP ratio. Bohn’s assumption is based on the fact that government budget is subject to changing circumstances and that governments usually react to increases in public debt instead of passively waiting to see their public debt evolving without putting some effort to control the debt level. This action is reflected in the policy response function, which has increasingly come to be known as the fiscal reaction function. A positive fiscal response means that the government takes action to reduce the deficit (or increase the surplus) when the public debt ratio rises. The fiscal reaction according to Born is specified as follows:

$$pb_t = \alpha + \rho d_{t-1} + z_t + \varepsilon_t$$

Where $\rho$ is the fiscal reaction parameter and measures the response of the primary balance to the lagged public debt ratio, $\alpha$ is the constant variable, $z_t$ is a set of other determinants of the primary balance and $\varepsilon_t$ is an i.i.d. error term shock to the primary balance. This specification shows, how, governments react to public debt accumulation given the structure of
macroeconomic shocks facing the economy. Substituting equation (6) into (5) and assuming that the non-fiscal determinants($z_t$) are zero result in the following equation.

$$\Delta d_t = (r_t - g_t - \rho)d_{t-1} - \alpha$$  \hspace{1cm} (7)

Equation (7) summarizes the determinants of public debt dynamics for the government, with the crucial factor being the sign of the term $(r_t - g_t - \rho)$ in front of the lagged public debt variable. If $(r_t - g_t - \rho) > 0$, implying that the interest rate exceeds the sum of the growth rate and the fiscal response coefficient, then public debt is intrinsically unstable. A rise in the public debt level, leads to a further acceleration of the growth of public debt, which means that debt is on a potentially explosive path. However, if $(r_t - g_t - \rho) \leq 0$, implying that the interest rate does not exceed the sum of the growth rate and the fiscal response coefficient, then the time path of debt is intrinsically stable.

The condition for stability $(r_t - g_t - \rho) < 0$ is referred to as the modified version of the Aaron condition, which is usually stated as the condition that the growth rate should exceed the interest rate, $(r_t > g_t)$ for government to have a sustainable public debt path. According to the modified Aaron condition, higher interest rates increase the growth of debt levels, whereas higher growth rates and a stronger responsiveness of the budgetary policy to debt tend to reduce debt growth.

Following Bohn’s literature, several authors have investigated the potential relationship between the primary balance and the public debt ratio. For instance, Hamilton and Flavin (1986), analysed the budget inter-temporal relation and No Ponzi game condition. This approach has been criticized by Bohn (2008) who proposed a sustainability test based on whether the primary balance to GDP ratio is a positive linear function of the debt to GDP ratio. Roubini (2001) showed that a stable debt to GDP in the medium to long term is considered as sustainable regardless of its level. According to him a debt to GDP ratio of 150 percent is as sustainable as a debt to GDP ratio of 50 percent. Ghosh et al. (2013) affirm that the sustainability indicator introduced by Bohn is too weak, and advocated for a new framework for assessing debt sustainability in the advanced economies by determining a debt limit beyond which fiscal solvency is in doubt.

There is also a general empirical premise that governments usually behave responsibly, increasing primary surpluses in response to rising debt service so as to stabilize the public debt-to-GDP ratio at a reasonable level. This empirical finding is consistent with the findings.
of Bohn (2008) for the US, and Mendoza and Ostry (2008) for subsets of industrial and emerging economies. Despite the existence of this vast theoretical and empirical literature on public debt sustainability, literature is far from being settled. There is no consensus on the optimal (stable) debt limit and corresponding fiscal space, which provide early warning guide to exploding public debt levels. This paper, thus applies the dynamic stochastic framework to simulate the public debt path for Zimbabwe given the nature of shocks facing the economy and its fiscal stance. This is particularly important to avoid the mistakes of accumulating public debt to unsustainable levels.

SECTION FOUR: RESEARCH METHODOLOGY

The paper applies the methodological approach of Bohn (1998, 2008) to analyse Zimbabwe’s debt sustainability using historical information. The methodology also draws from the method proposed by Celasun et al. (2006) which simulates interest rates and growth rates, taking account of the uncertain nature of these variables. The method combines the simulated interest rates and growth rates with the estimated fiscal response in order to determine the evolution in the public debt to GDP ratio. The analysis yields a distribution of possible time paths for future public debt to GDP ratios. This distribution can be characterized by the median for the public debt to GDP ratio and the confidence interval around this value.

First, the methodology consists of estimating the fiscal reaction function in line with Bohn’s specification. Accordingly, the primary fiscal balance, which is considered the key operational target of the fiscal authorities, was estimated as follows:

\[
pb_t = \alpha_t + \rho d_{t-1} + \gamma ygap_t + X_t \beta + \epsilon_t
\]  

(8)

where \(pb_t\) is the primary balance at time \(t\), \(\alpha_t\) is the intercept \(d_{t-1}\) is the public debt level at the end of the previous period, \(ygap_t\) is the output gap, \(\epsilon_t\) is an error term, and \(X_t \beta\) is a vector of macroeconomic variables explaining changes in the primary balance unrelated to the solvency requirement.

With respect to the expected coefficients from the fiscal reaction function, a zero or negative coefficient of the lagged debt to GDP indicates that governments fail to respond effectively, or even have a perverse reaction of increasing the deficit when debt increases. This is referred to as fiscal fatigue by Ghosh et al. (2013). The specification includes a range of other explanatory variables suggested by Celasun et al., 2006; Mendoza and Ostry, 2008; Burger et
al., 2011; and Ghosh et al., 2013). These variables are the output gap to control for cyclical fluctuations, and political and institutional variables.
4.1 Estimation technique

The paper applies the method of ordinary least squares (OLS) to estimate the primary fiscal reaction function. However, estimation of the fiscal reaction function raises some econometric issues stemming from the dependence of primary balance on past values of the debt to GDP ratio. A country able to generate higher primary balances on average would also tend to have lower levels of public debt. This negative association between debt and the primary balance, if not properly accounted for, could generate a downward bias in the estimated response of primary balance to lagged debt.

As such, the Hausman endogeneity test was applied to test for potential endogeneity between the primary balance and the output gap. For robustness check, the model was also estimated using panel data for selected Low Income Countries (LICs) in the MEFMI region. The estimated coefficients from the fiscal reaction function were then used to project the public debt path for Zimbabwe given its macroeconomic outlook as specified in the IMF World Economic Outlook (WEO), and the country’s macroeconomic Framework (IMF 2012).

4.2 Stochastic Simulation approach

The stochastic simulation was done in three stages, namely, estimating the fiscal reaction function, projecting the primary balance, using the variance of the residuals from the estimated fiscal reaction function and a standard normal distribution to generate the public debt path. The estimated fiscal reaction function from equation (8) above will be as follows:

\[ \hat{pb}_t = \alpha_t + \hat{\rho}d_{t-1} + \hat{\beta}_t\text{gap}_t \]  
(9)

The error term is computed from the estimated fiscal response function in equation (9) as follows:

\[ \varepsilon_t = pb_t - \alpha_t - \hat{\rho} * d_{t-1} - \hat{\beta}_t\text{gap}_t \]  
(10)

The error term is then specified as follows:

\[ \varepsilon_t = \phi\varepsilon_{t-1} + \mu_t \]  
(11)
Where $\mu_t \sim N(0, \sqrt{(1 - \varphi^2) * \sigma_e}$ and $\sigma_e$ is the estimated standard error of the regression. The effective interest on public debt was computed as a weighted average of domestic and foreign rates:

$$r_t = i_t^d d_{t-1}^d + i_t^f d_{t-1}^f$$

The weights of public debt denominated in foreign currency $d_{t-1}^f$ and domestic debt $d_{t-1}^d$ followed the historical pattern and was varied to reflect the government’s thrust to develop the domestic debt market in the medium to long-term. Projections of the underlying public debt dynamics were obtained through a fiscal reaction function and forecasts of the macroeconomic variables were obtained from Zimbabwe’s macroeconomic policy framework (IMF, 2012).

### 4.3 Baseline Public Debt to GDP Simulation

The estimated fiscal response function was used to simulate the public debt path together with assumed growth rate of the economy and simulated interest rate structure from the macroeconomic framework. Replacing the primary balance with the estimated primary balance reaction function in equation (9), results in the following simulated public debt position:

$$d_t = \varphi d_{t-1} - (\alpha_t + \hat{\rho} d_{t-1} + \hat{\beta}_t \text{gap}_t)$$

(15)

This translates to the following equation after collecting the like terms.

$$d_t = (\varphi - \hat{\rho}) d_{t-1} - (\alpha_t + \hat{\beta}_t \text{gap}_t)$$

(16)

Equation (16) implies that the future public debt path is determined by automatic debt dynamics $\varphi$ and the estimated fiscal response parameter $\hat{\rho}$ and non fiscal determinants of public debt policy which in this case is the output gap. The simulations were conducted under various scenarios on growth rate given the simulated interest rate path and the estimated fiscal policy response. As previously stated the simulated public debt path can only be sustainable if the modified Aaron condition is satisfied, implying that the effective interest rates on public debt do not exceed the sum of growth rate and the fiscal response on average. This implies that the debt will revert to an equilibrium steady state after a macroeconomic shock.
4.4 Bounds on the Public Debt Limit

In order to derive the range of the sustainable public debt path, the upper and lower bound of public debt dynamics were also established. These bounds provide an insight into the range within which the optimal public debt target would lie. As such, in line with the approach taken by Ghosh et al (2013), the upper bound $d_2$ was computed as the largest root of the following equation:

$$\alpha + \rho(d_2) + \bar{\epsilon} = (r^* - g_t)d_2$$  \hspace{1cm} (17)

The left-hand-side of equation 17 shows the best primary surplus that the government can achieve at a debt ratio $d_2$ under the best realization of macroeconomic shocks. The right-hand-side shows the lowest effective interest payment required by government to reduce or mitigate the risk of default. As public debt increases beyond $d_2$, the primary surplus would increase at a slower rate than $(r^* - g_t)$. As a result, if the debt ratio exceeds $d_2$, the primary balance would not suffice even under the best of circumstances to cover the interest payment, and debt dynamics would become explosive, thus, violating the modified Aaron condition and triggering default. Similarly, the lower bound, $d_1$ was obtained as the largest root of the following equation:

$$\alpha + \rho(d_1) - \bar{\epsilon} = (r^* - g_t)d_1$$  \hspace{1cm} (18)

The left side of equation 18 shows the smallest primary surplus required to cover the effective interest payment to ensure sustainability. In this case, since debt is non-increasing at $d_1$ for all possible realizations of macroeconomic shocks, and there is no risk of default. The lower limit, thus correspond to the so called natural debt limit, defined in the macroeconomic literature on savings under incomplete markets as the largest debt that the government could take if it wants to ensure that it will never default in the next period or at any point in the future, even if the primary balance remains at its worst realization forever. The results were expressed in terms of a fan chart and distributions.
SECTION FIVE: RESULTS AND ANALYSIS

This section provides the results of the fiscal reaction function, together with the simulated public debt-to-GDP ratio for Zimbabwe. The results are used to determine the probability that debt will exceed the desired SADC regional indicative target of 60 per cent in the medium to long term. The analysis also facilitated determination of the upper and lower limit of the public debt, which are critical in establishing the natural debt limit and the point at which debt becomes highly unsustainable.

The stochastic simulation results are based on Zimbabwe’s macroeconomic framework for 2013 and the medium term. The IMF (2012) macroeconomic framework for Zimbabwe, forecast the country’s real GDP to grow at an average of at least four per cent over the medium term driven mainly by increased investment in mining and vigorous growth in construction, electricity and manufacturing as the business environment improves. The framework suggests that the country’s inflation rate would remain contained at an average of about 4.5 per cent in the medium to long term. On the fiscus, the government is projected to generate cash surpluses averaging 1.5 per cent over the medium term, mainly reflecting higher diamond revenue, a declining wage bill, reflecting payroll control measures and containment of wage increases at or below inflation. This would enable a more sustainable fiscal path allowing for the rebuilding of fiscal buffers and the international reserves, while improving service provision.

5.1 Estimated fiscal reaction function

The estimated fiscal reaction function only considered the fiscal determinants of public debt and omitted other determinants of the primary balance. These factors were omitted to avoid determining their out of sample trajectories required to simulate public debt paths until 2020. All the variables are integrated of order 1 and the signs of the main explanatory variables are as expected. The Hausman endogeneity test, also accepts the null hypothesis of weak instruments and as a result the ordinary least squares methodology was applied to estimate the fiscal reaction function for Zimbabwe. For the panel data for MEFMI LICs, the Hausman Specification test was used to select the fixed effects or random effects model. The test chose the random effects as a perfect fit. As a result, the results of a random effects model which jointly captures cross-country and within-country determinants of primary balance were used. The results from the estimated fiscal reaction function are shown in Table 5.1 below.
Table 5.1: Primary balance reaction function

<table>
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<tr>
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<th>ZIM: Model 1</th>
<th>MEFMI: Model 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-0.081***</td>
<td>-0.043***</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.012)</td>
</tr>
<tr>
<td>Lagged Debt/GDP Ratio</td>
<td>0.072***</td>
<td>0.064***</td>
</tr>
<tr>
<td></td>
<td>(0.014)</td>
<td>(0.013)</td>
</tr>
<tr>
<td>Output Gap</td>
<td>-0.085*</td>
<td>-0.027**</td>
</tr>
<tr>
<td></td>
<td>(0.074)</td>
<td>(0.056)</td>
</tr>
</tbody>
</table>

**Diagnostic Test**

<table>
<thead>
<tr>
<th></th>
<th>ZIM: Model 1</th>
<th>MEFMI: Model 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adjusted R-Squared</td>
<td>0.412</td>
<td>0.485</td>
</tr>
<tr>
<td>Durbin-Watson Stat</td>
<td>1.627</td>
<td>2.372</td>
</tr>
<tr>
<td>Hausman Test</td>
<td></td>
<td>Prob &lt; chi2 = 0.243</td>
</tr>
</tbody>
</table>

Source: Researcher’s own computations, Note: ***, significance at 1%, **, significance at 5%, *, significance at 1% and Figures in parenthesis are p-values

The results from Table 5.1 show that the primary balances respond positively to increases in the public debt-to-GDP ratio, suggesting that the inter-temporal budget constraint exists. The output gap has a negative sign indicating that the primary balance response negatively to increases in the output gap. This would indicate a pro-cyclical fiscal policy. The results show that if the public debt to GDP ratio improves by 1% of GDP in year t, the primary balance would improve by 0.7% of GDP in year t+1. The insignificant coefficient of the output gap in ZIM Model 1 shows that Zimbabwe’s fiscal policy position has been a-cyclical during the three decades, from 1980 to 2012. Nevertheless, the coefficient for the output gap for low income countries in the MEFMI region MEFMI Model 2 is negative and significant indicating a pro-cyclical fiscal policy stance. This result is consistent with previous research that found fiscal policy in low income countries to be pro-cyclical. This contrasts results from fiscal policy in high-income countries where it is usually found to be countercyclical (Ilzetzki & Vegh, 2008). Burger et al. (2011) found a positive and statistically significant coefficient for South Africa indicating that the country follows a counter cyclical fiscal approach.
5.2 Stability of the fiscal reaction function

The structural stability of the results from the fiscal reaction function was examined using the Cumulative Sum of Recursive residuals (CUSUM) and the CUSUM of square (CUSUMSQ) tests proposed by Brown et al. (1975) to assess the parameter constancy. The results are shown in Figure 5.1 below.

**Figure 5.3: Stability of the reaction function**

![CUSUM and CUSUMSQ plots](image)

*Source: Researcher's own computations*

In Figure 5.1, the results from the CUSUMSQ and CUSUM show that the estimated fiscal reaction function for Zimbabwe is stable. The results indicate the absence of any instability of the coefficients because the plot of the CUSUMSQ and CUSUM statistic are confined within the 5 per cent critical bounds of parameter stability. The results can, therefore, be safely applied to make stochastic simulations of public debt projections.
5.3 Implications for public debt management

Combining the information on interest rate and growth rate together with the estimated coefficients from the fiscal reaction function provides information for testing the Aaron condition. Exogenous shocks in the interest rate/growth differential can cause shifts from a stable to unstable public debt equilibrium. Figure 5.2 below shows the historical movement in the gap between the fiscal response parameter ($\hat{\rho}$) and interest and growth rate differential ($\emptyset$).

**Figure 5.2: Historical movements in ($\emptyset - \hat{\rho}$) gap**

![Graph showing historical movements in ($\emptyset - \hat{\rho}$) gap](image)

Source: Researcher’s own construct

The results in Figure 5.2 show that Zimbabwe only satisfied the modified Aaron condition ($\emptyset - \hat{\rho} < 0$) from 2009 onwards wherein the growth rates exceeded interest rates on average. This reflects the dominance of concessional external loans in the public debt portfolio in the absence of significant domestic debt borrowing. Since the introduction of the multiple-currency system, Zimbabwe has not been able to borrow significant amounts from the domestic debt market due to attendant liquidity risks. During the crisis period, Zimbabwe experienced a significant decline in real growth rates and a steady rise in real interest rate/growth differential on the external front. These factors resulted in an explosion of public debt.
debt. The violation of the Aaron condition over the period 1980 to 2009 implies that solid fiscal responses were needed for government to ensure sustainable public debt. The introduction of the cash budgeting system by the Zimbabwean government in 2009 was, therefore, a positive response and partly contributed to alleviating the public debt burden.

5.4 Uncertainty on public debt projections
The need for fiscal reaction functions points to the need to assess whether governments are really in control of their debt. This is because debt dynamics are uncertain in nature and depend on macroeconomic shocks that affect the economy. The results of the stochastic framework, thus, provide insight into the evolution of debt under alternative macroeconomic shocks. Accordingly, using information from the estimated fiscal response function, 5 000 dynamic stochastic public debt simulations were conducted over the period 2012-2020. The simulations were based on observed behaviour of interest rates and growth rates until 2012.

The output gap was obtained from the Hodrick Prescott fitter and inserted into the debt dynamics equation to obtain the public debt path. The construction of the confidence intervals for the public debt-to-GDP ratio follows simulation methods on debt sustainability (Garcia & Rigobon, 2004 and Mendoza & Oviedo, 2004). The simulation methodology follows two steps. The first step involves the extraction of the co-variance structure of shocks. The second step involves running a Monte Carlo simulation which feeds these shocks into the public debt dynamics. The co-variance of shocks to real GDP, interest rate and primary balance were extracted from historical data over the period 1980 to 2012. The results are shown in the Table 5.2 below.
Table 5.2: Variance Covariance Matrix of Shocks

<table>
<thead>
<tr>
<th></th>
<th>Growth</th>
<th>Public Debt</th>
<th>Primary Balance</th>
<th>Output Gap</th>
<th>Interest Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Growth</td>
<td>0.005</td>
<td>-0.004</td>
<td>0.000</td>
<td>0.003</td>
<td>0.001</td>
</tr>
<tr>
<td>Public Debt</td>
<td>-0.004</td>
<td>0.052</td>
<td>0.006</td>
<td>-0.017</td>
<td>-0.007</td>
</tr>
<tr>
<td>Primary Balance</td>
<td>0.000</td>
<td>0.006</td>
<td>0.004</td>
<td>-0.004</td>
<td>-0.001</td>
</tr>
<tr>
<td>Output Gap</td>
<td>0.003</td>
<td>-0.017</td>
<td>-0.004</td>
<td>0.043</td>
<td>0.005</td>
</tr>
<tr>
<td>Interest Rate</td>
<td>0.001</td>
<td>-0.007</td>
<td>-0.001</td>
<td>0.005</td>
<td>0.002</td>
</tr>
</tbody>
</table>

Source: Researcher’s own computations

The variance covariance results in Table 5.2 shows a negative relation between public debt and economic growth. This is consistent with literature. The only perplexing result is the negative relation between interest rates and public debt. The result, however, suggests that government resorts to debt consolidation when interest rates increase. A distinctive advantage of the methodology based on the historical variance-covariance matrix lies in the possibility to use model-independent forecasts to define the baseline scenario around which macroeconomic shocks are applied (Berti, 2013).

5.5 Simulated baseline of public debt dynamics

The stochastic simulations facilitated a derivation of a comprehensive indicator for assessment of a sustainable public debt policy. The simulations were undertaken by assessing the ‘at risk’ indicator, which measures the degree to which governments are in control of their public finances. The indicators were constructed as the median debt path in the simulation and the expected probable upper and lower bound for the debt level in the medium term. In the literature on stochastic public debt, the distribution of public debt is summarised using fan-charts (Meridios, 2013). The fan chart provides a probabilistic view of the uncertainty around the baseline by showing a spectrum of possible outcomes. It also facilitates the computation of the stochastic properties of the data that incorporates the interaction between macroeconomic variables (Celasun et al. 2006).
Accordingly, the results were summarised in a fan-chart type of analysis, which provided the frequency distribution of the calibrated public debt paths and served to illustrate the overall range of risks to the public debt dynamics. Fan-charts also provide a probabilistic view of the uncertainty around the baseline (IMF, 2013). The results are shown in Figure 5.3 below.

Figure 5.3: Fan Chart: Simulated public debt dynamics

The results from the baseline stochastic simulation in Figure 5.3 show a general decline in public debt-to-GDP ratio for Zimbabwe in the medium to long-term. The results indicate that Zimbabwe’s public debt would be slightly below the 60 per cent mark by 2020. The results, however, show a great deal of uncertainty in the projected debt-to-GDP ratio. The stochastic distribution shows the public debt-to-GDP ratio for Zimbabwe to be around 60 per cent for all the scenarios, with a lower bound of approximately 40 per cent of GDP. The lower bound can be regarded as the natural debt limit which is the debt level that the country can accommodate without fearing the risk of default, even under extreme macroeconomic shocks (Ghosh et al. 2013). Furthermore, the results show the ‘at-risk’ indicator of about 30 per cent.
of GDP. The at-risk indicator, measures the deviation of the upward dispersion in the simulation. It is computed as the expected probable upper bound minus the expected debt level in the medium term projection. The median projections show that Zimbabwe’s public debt would not get out of control until the end of the forecasting horizon. As a result, Zimbabwe’s public debt can thus be regarded to be sustainable from an inter-temporal solvency condition over the period from 2013 to 2020.

5.6 Distribution of the simulated public Debt-to-GDP ratio
The theoretical distributions shown in Figure 5.4 below confirm that Zimbabwe’s public debt follows a normal distribution. A skewed distribution would imply more risk to future debt sustainability. Hall (2013) shows that if the distribution of the simulated debt paths is properly defined in the long-run, the debt-to-GDP ratio is stationary and follows a near unit root instead of a unit root process.
Figure 5.4: Distributions of the simulated stochastic public debt path

Source: Researcher’s own computations based on E-views

The distributions in Figure 5.4 capture interactions among the macroeconomic and fiscal variables being shocked, and informs about the plausible range of risks associated with the projected public debt paths. This in turn, prepares policy makers for a better-informed policy reaction should such risks materialise. The distributions are also informative of public debt sustainability. A narrower distribution indicates greater certainty on future debt dynamics and characterises a country that is more in control of its finances. The distributions show that the debt level is skewed towards high debt ratios despite the shocks being normal. This implies that the snowball effect grows with the level of public debt. The distribution for the baseline
scenario mean shows a slightly wider debt-to-GDP from as low as 40 to 160 per cent. The lower bound, however, shows a narrow distribution of up to 50 per cent. This implies that Zimbabwe would require a debt-to-GDP within this range to guarantee medium to long-term sustainability.

5.7 Simulated probability of public debt burden

The stochastic simulation analysis also facilitated the computation of the probability of the public debt burden in the medium to long-term. The probability was computed as the likelihood that public debt would exceed the median debt limit and the computation is illustrated in equation 19 as follows:

\[ p_{t+1} = pr[d_{t+1} > \bar{d}] \]  

(19)

Where \( p_{t+1} \) the primary balance is in the next period, \( d_{t+1} \) is the period ahead public debt to GDP in the following period, \( \bar{d} \) is the debt limit and \( pr \) the probability. The simulation results suggest a probability of about 47 per cent for the debt ratio to exceed the 60 per cent debt threshold under the baseline scenario and alternative scenario as shown in Figure 5.5 below.

Figure 5.5: Probability of public debt value exceeding 60% in 2020 (Baseline Scenario)
The results suggest that there is a higher probability that Zimbabwe will be able to maintain its debt within sustainable limits, given its fiscal stance and the nature of macroeconomic shocks facing the economy. The probability estimates are necessary to flag instances where the government may be required to undertake fiscal consolidation. Moreover, it ensures that public debt remains on a sustainable path to ensure that macroeconomic shocks do not derail public debt from its predetermined optimal path.

5.8 Diagnostic tests
The diagnostic tests for the stochastic simulation discussed in the foregoing were undertaken using the quintile to quintile (QQ) plots. The results for the baseline scenario are shown in Figures 5.6 below.
The results from the normal Q-Q plots in Figures 5.6 strongly support the view that the public debt distributions are normal. The results further show that the actual observations, the blue dots, are slight deviations from the red line. This confirms the stochastic model’s forecasting ability.
SECTION SIX: CONCLUSION AND POLICY RECOMMENDATIONS

This section provides a summary of the issues raised in the paper and policy implications. The main contribution of the stochastic approach is that it points out how important uncertainty is to public finances, and how important proper fiscal institutions are in dealing with this uncertainty. The paper followed the methodological approach of Bohn (2008) and identified three channels that contribute to sustainable public debt. These channels are the economic growth, real interest payments and fiscal responses. The analysis combined the estimated fiscal response with a stochastic debt simulation to create the median public debt path. The methodology applied involves estimating a fiscal reaction function and using it to simulate public debt dynamics by applying a stochastic approach and historical information on drivers of public debt accumulation and their volatility.

The stochastic model can assist in assessing the factors that contribute to the fiscal challenges. The analysis can also be used to flag the structural adjustments required to ensure sustainable public debt dynamics. The stochastic simulation analysis also provided the probability indicators for the potential future public debt burden. This assists in assessing the susceptibility of the country to adverse macroeconomic shocks. The median public debt projections, in conjunction with the confidence intervals around them, illustrate the high risks associated with public debt dynamics in Zimbabwe. The results from the baseline scenario show that Zimbabwe’s public debt would not deviate much from the desired indicative public debt benchmarks in the medium to long-term. The risk to the public debt projection is, however, high with a 47 per cent probability of public debt exceeding the desired threshold in the medium to long term.

The analysis is consistent with the envisaged growth path for the economy of at least four per cent in the medium to long-term. The joint assessment of fiscal policy with public debt dynamics ensures that public debt simulations reflect both the direct impact of the primary balance on reducing the debt, as well as its indirect impact through macroeconomic fundamentals. The results from the fiscal reaction function show that the budgetary policy reacts positively to lagged debt and negatively to the contemporaneous lagged output gap. If the past is a guideline to the future, the median projections indicate that the public debt path for Zimbabwe would follow a sustainable path from an inter-temporal solvency condition.
until the end of the forecast horizon. The probabilistic sustainability indicator revealed a high probability of 53 percent for a sustainable debt in the medium term from an inter-temporal solvency condition.

The stochastic simulation presented in this chapter does not, however, take into account the liquidity risks or changes in public debt due to the recognition of contingent liabilities. Since the results show a sustainable public debt position from a solvency perspective, future research should explore the implications of liquidity or rollover risk on public debt dynamics. From a policy perspective, the results of the stochastic simulation provided in this chapter are necessary to flag instances where fiscal consolidation may be needed. The implication for public debt management is the need for policy makers to proactively respond to the changing macroeconomic environment and to implement counter-cyclical fiscal policies to limit the probability of public debt from exploding.

REFERENCES


Medeiros, J. (2012). Stochastic debt simulation using VAR models and a panel fiscal reaction function: results for a selected number of countries. European Commission


